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A Case Comparative Study on Open Mathematical Modeling between Gifted and Ordinary High School Mathematics Students

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Abstract: ***This study selected a representative gifted student and an ordinary student as case subjects, and explored the performance differences between the two in the process of open mathematical modeling through observation, work analysis and other methods. Based on Leiß and Blum 's seven-step cycle schema for mathematical modeling, a comparative study of open mathematical modeling cases was conducted. The study found that gifted students can quickly understand the meaning of the questions, convert the questions into sub-questions to build a reasonable model framework, and have flexible thinking in the modeling process, and can quickly call on existing mathematical knowledge; while ordinary students have certain difficulties in understanding the problems, are prone to give up when encountering obstacles, and are not flexible in the application of mathematical knowledge ; both of them perform poorly in testing and adjusting the model. Through comparative analysis, the different characteristics and gaps between the two in the development of mathematical modeling ability are revealed, which provides a reference for teaching students in accordance with their aptitude and improving the mathematical modeling literacy of students at different levels, and helps high school mathematics education better meet the personalized development needs of students.***

Keywords: Gifted Students and Ordinary Students; Seven-step Cycle Schema for Mathematical Modeling; Open Mathematical Modeling; Case Study.

1. INTRODUCTION

Discussions on the integration of applications and modelling in mathematics education have been taking place since the beginning of the century [1][2], and mathematical modelling is now part of many mathematics curricula around the world [3-5]. However, each country has its own historical, socio-cultural context that has shaped its own school mathematics education. Although interest in international comparisons of different aspects of mathematics teaching has increased significantly over the past few decades, there are few research results comparing the impact of different educational traditions on specific mathematical activities in different European countries [6], and even fewer specifically on mathematical modelling. Mathematical modelling is defined as the application of mathematical means to solving real-world problems, i.e., translating real-world situations into mathematical forms, working in a constructed mathematical model, and interpreting the obtained mathematical results [7]. For example, Leiß and Blum divided the mathematical modelling process into seven steps [8], which together form the so-called modelling cycle. These seven steps are: understanding the real situation and building a mental model, structuring and simplifying the mental model, mathematizing the real-world model, working in a mathematical model, interpreting the mathematical results, verifying the results, and proposing solutions. Mathematical modelling ability is usually defined as the ability of an individual to build and use mathematical models to solve real-world problems and to analyse or compare given models [9]. Open mathematical modeling problems are open tasks based on complex real-world situations. Their essence lies in systematically reconstructing and solving unstructured real-world problems through mathematical tools. Such problems have significant uncertainty characteristics and require modelers to go through a complete cognitive cycle from abstracting real-world situations to mathematical representation and then returning to practical verification. They play a unique role in improving students' mathematical modeling ability [10].

Mathematical modeling and its applications are important components of the curriculum and are considered to be crucial to students' current and future lives [11]. Focusing on mathematical modeling from a cognitive perspective can help conduct micro-analysis in this area. Since "in-depth, small-scale comparative studies can provide us with unique opportunities to understand students' mathematical thinking" [12], comparing two students with different mathematical abilities can help us generate hypotheses and enhance our understanding of why mathematical modeling is often so difficult for students [13]. Therefore, this study uses a qualitative research method to conduct a case study on the cognitive barriers faced by high school students in open mathematical modeling, and analyzes and compares the cognitive barriers of individuals based on the SSCS (Seven-step Cycle Schema for Mathematical

Modeling) open mathematical modeling schema developed by Blum et al.

2. RESEARCH QUESTIONS

(1) In a specific open-ended mathematical modeling situation, what different cognitive paths do gifted high school students and ordinary students present when facing an open-ended mathematical modeling task? What obstacles do the two have when conducting open-ended mathematical modeling?

(2) Based on the SSCS theory, in previous experience, what kind of connections or differences exist between gifted high school students and ordinary students in their cognition when facing the same open-ended mathematical modeling challenge?

3. RESEARCH METHODS

In order to find out what kind of cognitive schema deconstruction and reconstruction process middle school students present when doing open mathematical modeling tasks, and what kind of cognitive differences exist between mathematically gifted students and ordinary students in previous experience when facing the same modeling challenge, this study first selected two typical mathematically gifted students and ordinary students in the same class of the same high school, and the two participated in this research link on a voluntary basis. The researchers gave the two high school students an open mathematical modeling task in an isolated environment. After receiving aloud thinking training, the students used the aloud thinking method to answer open mathematical modeling questions. When the students encountered a situation where they could not proceed, the researchers would give necessary prompts, and the whole process was recorded. Afterwards, the thinking steps of the two high school students were coded and timed accordingly through the video, and then the SSCS was used to depict the modeling path of middle school students when doing open mathematical modeling tasks, so as to further compare and analyze the cognitive paths shown by the two students when they first did open mathematical modeling.

4. STUDY SUBJECTS

4.1 Selection of Research Subjects

This study adopted a convenient sampling method and selected two first-year high school boys from a key middle school in Zibo City, Shandong Province, China, who were in the same class and environment, as case study subjects. Through comparative analysis, the cognitive and emotional factors affecting the differences in mathematics performance were explored. Although the research subjects, Wang and Li, were in the same educational environment, their mathematics performance showed significant differentiation: the former's mathematics performance has long been at the top of the class and grade, showing strong logical thinking and learning initiative; the latter's mathematics performance is at the middle and lower levels, with insufficient interest in the subject and defects in learning methods. It is worth noting that both students had just completed the first semester of their first year of high school when they participated in the study, and had no previous exposure to open mathematical modeling activities or special competition training. By analyzing the heterogeneous performance of typical cases, this study attempts to reveal the deep cognitive mechanism that affects the effectiveness of open mathematical modeling, in order to provide empirical evidence and practical inspiration for optimizing mathematical modeling teaching strategies and improving students' modeling literacy.

4.2 Sketch of the Research Object

4.2.1 Student Wang

The subject of this study, Mr. Wang, is 15 years old. His total score at admission ranked 8th in his class and among the top 100 in the district. Since junior high school, his advantage in mathematics has been very prominent. The student's math scores in junior high school have been consistently in the top 5% of the grade. After entering high school, he still maintains the leading level of the top three in the class and the top 5% in the grade. His mathematical cognitive development shows distinct structural characteristics: in terms of thinking style, Wang has built a complete logical reasoning framework. He can not only build a logical chain for solving problems in a clear and orderly manner, but also knows how to double-check the steps of solving problems to ensure that the answers are correct; in terms of learning attitude and behavior, Wang has shown extremely high concentration and is often immersed in the "flow" state of learning. His way of sorting out wrong questions is also very special. He does not simply correct the mistakes, but will mark in detail the points where his thinking is stuck when solving problems, and come up with other possible solutions. At the same time, he has a strong ability to monitor his own thinking process and can clearly feel the difficulties he encounters when switching between geometric proofs and algebraic operations; in terms of knowledge structure and ability, Wang can easily deal with complex mathematical variable relationships. His strong sense of self-supervision also ensures that when establishing mathematical models, he can continuously adjust and optimize according to actual conditions. Wang's multi-dimensional cognitive advantages work together, indicating that he may perform well when facing open-ended mathematical modeling tasks and be able to flexibly switch problem-solving ideas. His cognitive development process provides a good example for studying how to cultivate excellent mathematics students.

4.2.2 Student Li

Student Li is 15 years old this year. He is a classmate of Student Wang. The two sit at the same table, so they have a good relationship. When he entered school, his total score ranked 41st in the class and 1055th in the district. In fact, his academic performance was not very stable in junior high school, and the fluctuation of his grades had a lot to do with his performance in mathematics. He said that his junior high school entrance examination score was the worst since junior high school, and mathematics became the key factor that dragged down his overall performance. In the four exams in the first semester of the first year of high school, his total score ranked in the top half of the class (a total of 50 people in the class), but his mathematics score was always ranked in the bottom 15%, and the gap between the two subjects was particularly obvious. Student Li obviously encountered obstacles in cognitive development when learning mathematics. He ranked between 35th and 40th in the class for a long time, and there was an obvious shortcoming in his mathematical ability: his grades had not improved, and it felt difficult to improve. He always applied fixed methods when solving problems, was not very flexible, and had shortcomings in understanding the connection between mathematical concepts. His math learning style is rather scattered, and the knowledge points in his notebook are very messy and have not formed a systematic connection. In addition, Li has put a lot of effort into math, but the effect is not good. Compared with other subjects, he spends 40% more time on math homework, but the accuracy rate is only 58%. This low efficiency makes him more and more afraid and anxious about math. He himself said that when doing math problems, he often "doesn't know what he is doing and what to do next", which shows that he lacks effective self-monitoring in mathematical thinking. Li's unstable cognitive structure indicates that he may encounter considerable challenges in knowledge application and overall thinking in open modeling tasks. His cognitive development process provides a good example for us to understand the obstacles that ordinary students face in open mathematical modeling.

5. MATERIAL

5.1 Test Questions

The staff in the main control room of the power plant mainly operates and controls the generator according to the data changes of the instrument. The bottom edge of the instrument is 2 meters from the ground. The staff stands to look at the instrument. Where is the best place for the staff to stand?

Figure 1: Open mathematical modeling task - "Determination of the operating location of the power plant control room"

5.2 Cognitive Behavioral Coding Table

Table 1: Open mathematical modeling coding table based on SSCS

Step	Operational Definition	Typical Behavioral Markers
Understanding real-world situations	Repeat the key words in the question to clarify what needs to be done	Point your finger to the problem statement area eg. "Actually use this part right here"
Idealization, Structure, Simplification	Writing Constraint Lists	The appearance of "assuming..." type statements Eg. "Yes, we need to assume the height of the staff"
Abstraction as a mathematical modeling problem	Solve problems with mathematical methods	Symbol system appears in the answer area Eg. "The observation angle needs to be the largest"; "Use the method of combining numbers and shapes to express this problem situation with images"
Building a mathematical model	mathematical procedures followed	List mathematical formulas and perform calculations eg. "Next we need to calculate tan(α－γ)"
Explaining Mathematical Results	Refer to the details in the context of the question	Consider units in mathematical solutions; contextualize mathematical work; provide quantitative data or qualitative indicators of results Eg. "The calculation result shows that the staff should stand 0.25 meters away from the instrument"
Test	Check the rationality of the process or method	Check assumptions and results from previous experience and methods Eg. "You should be able to see clearly from this position"

6. TWO STUDENTS' OPEN MATHEMATICAL MODELING COGNITIVE PATHS

6.1 Student Wang

"Teacher, is this the only question?" Wang found that this question was different from the math test papers he had done before when he first received the question. Later, the researcher introduced to him that this was a mathematical modeling problem with a real-life situation, but did not point out the openness of this question. When reading the question, Wang circled the key information in the question while reading it. He focused on the two stem information "the instrument panel is 2 meters from the ground" and "best", thinking that this might be the key to solving the problem, but then he said, "Teacher, is this the only information in this stem?" He began to pay attention to the missing information necessary for solving this problem, but he didn't know what to do next, so he began to fall silent and chose the option "maybe not" in the task-based self-efficacy question. After being silent for more than 30 seconds, he said, "There may be something wrong with this question... It has no answer, right?" The researcher then explained to him, "This is an open-ended question, so we need to make reasonable assumptions about the actual information." Wang then reread the question to understand it and said, "So in this question, I think the missing information is the height of the staff member and the size of the dashboard. Suppose his height is 1.8 meters and the dashboard is 1 meter..."

"I need to clarify what the core of the problem is, that is, to clarify what 'where is the best place to stand' means." So he fell silent again because of thinking. After about 35 seconds of silence, he began to continue to use the thinking aloud method to express his thinking, "It must not be the closer the better, because the closer the dial is, the things on the dial cannot be seen clearly. If I want to see it clearly and comprehensively, I may need to draw a picture to represent this scene..." So he first tried to draw two sketches on the back of the test paper to represent the situation. Later, he felt that neither of the two sketches was the best position, so he made several modifications on the two pictures. The whole process took more than two minutes, and he began to fall silent again. The researcher began to prompt him to try to find "the position with the largest observation angle". Wang showed a confused look and asked the researcher, "Teacher, why is it when the angle is the largest?" "Because we first want to observe comprehensively, we need to make two rays from the human eye pass through the upper and lower edges of the instrument panel respectively, but we also need to let people get as close to the instrument panel as possible, so that they can see more clearly, and then the observation angle is the largest at this time..." The researcher replied. After thinking for a moment, Wang drew a final sketch below the question on the front of the test paper and marked the picture with the letters needed to solve the problem.

"Make EC perpendicular to AD at point C, let FD=EC=x", here, Wang constructed a right triangle, and regarded the height of the instrument from the ground as BD, then he thought of the need to use the trigonometric function two angles sum and difference formula. He regarded the degree of ∠AEB as θ, and the degree of ∠BEC as α, so he chose the formula tan(θ+α). "I expanded this and got... and then continued to simplify and move the terms, and got tanθ equal to..." He is very proficient in the formula, and his thinking fluency in mathematical operations is very good. In addition, Wang's choice of mathematical tools is relatively flexible and accurate. When he simplified the final expression, after a brief observation, he thought of using basic inequalities to find the maximum value.

In the end, Wang gave a precise result based on mathematical calculations, without considering how much help this result would provide for real-world problems, nor did he try to verify the rationality of his answer. Instead, he immediately stated that he had finished answering the question. The entire process took him 13 minutes and 23 seconds.

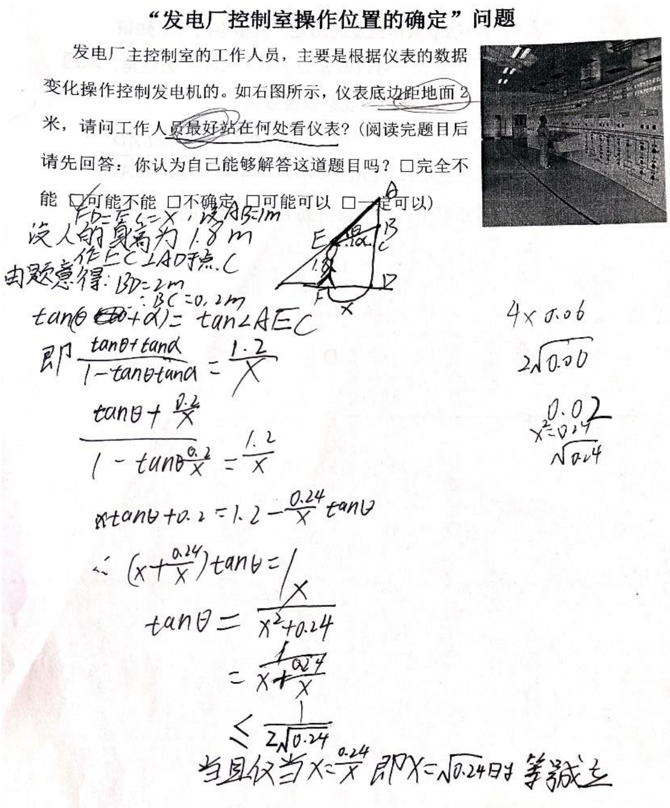

Figure 2: Wang’s open-ended mathematical modeling answer

6.2 Student Li

"There are so few questions!" Li was amazed when he first saw the open-ended mathematical modeling question. "How can I do this..." He showed a doubt about the question itself. Then, the researcher explained to him that this was a mathematical modeling question with a real-life situation, which was different from the application questions we had done on math papers in the past. The hints given by the researchers here were basically the same as those given to Wang, without indicating the openness of this question. So, he showed a certain interest in the question and began to read the question carefully. "We know that the bottom edge of the instrument is 2 meters from the ground... Teacher, what does it mean that it is best to stand where to see the instrument? Isn't it the closer the better..." The researcher guided him to think for a while. After falling into silence for nearly 20 seconds, he said, "It's not the closer the better. Maybe it needs some distance, because it's not clear if it's too close..." Then he chose the option "Maybe" in the task-specific self-efficacy question.

It didn’t take long for Li to find that the question lacked the necessary information to solve it. “This question lacks conditions… There is too little known information!” He began to question the question itself. So the researcher explained to him that this mathematical modeling problem is open-ended. “One characteristic of open-ended mathematical modeling problems is that they lack the necessary information to solve the problem. There is no standard answer to this type of question. We can make reasonable assumptions about the lack of information.” After Li received the prompt to make assumptions, he began to think about what information he needed to solve this problem. “I think I need to know how tall this staff member is. Well… I think the width of the dashboard may also need to be taken into account.” From this, it can be seen that Li is very sensitive to the information in the question, and his search for missing information is relatively accurate and comprehensive.

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"Assume that the height of the staff is 1.7 meters, and the width of the instrument panel is half a meter... Next, I need to consider where to stand." However, he fell silent soon after. He began to use sketches on the draft paper to try to abstract the real situation into a mathematical problem. During this period, he failed to express his ideas well by thinking out loud. So the researchers used teaching prompts to help him overcome this thinking obstacle. "Here we need to find the position where the staff can observe the largest angle, because we first want to observe comprehensively, so we need to make two rays from the human eye pass through the upper and lower edges of the instrument panel respectively, but we also need to let people get as close to the instrument panel as possible, so that they can see more clearly, and then the observation angle is the largest..."

Student Li spent about 50 seconds drawing a sketch on the test paper that could answer the question according to the researcher's prompts, and marked the information needed to solve the problem in the sketch. "The degree of ∠ABC is α, and the degree of ∠DBC is γ. In order to ensure that the staff can observe clearly and comprehensively, α-γ needs to be as large as possible... No, it should be moderate." The researcher was not allowed to interfere with the student's entire thinking process, and Student Li continued to think. He first tried to use tanα and tanγ to represent BC, but found that this was useless for solving the problem. After being silent for about 25 seconds, he thought of the two angles and difference formula of trigonometric functions. Then Student Li wrote the formula tan (α-γ) in the answer area of the test paper and then started to calculate. However, after simplifying the formula, Li was silent again for nearly a minute, and then he said, "I don't know how to find the minimum value." The researchers then explained to him that "α-γ should be as large as possible" and guided him to actively recall the methods of finding the maximum value he had learned in the past. Finally, he thought of using basic inequalities to find the maximum value.

After the prompt, Li wrote out the remaining math steps smoothly, and made some estimates for the results with square roots, and did not use the exact answer obtained by calculation as the final answer to the question. However, he was careless and made a mistake in the calculation, and the final result should be approximately equal to 0.5m instead of 0.25m. However, it is worth affirming that he finally reviewed and tested the result he obtained and believed that this result was relatively reasonable. The whole process of Li's answer took a total of 15 minutes and 21 seconds.

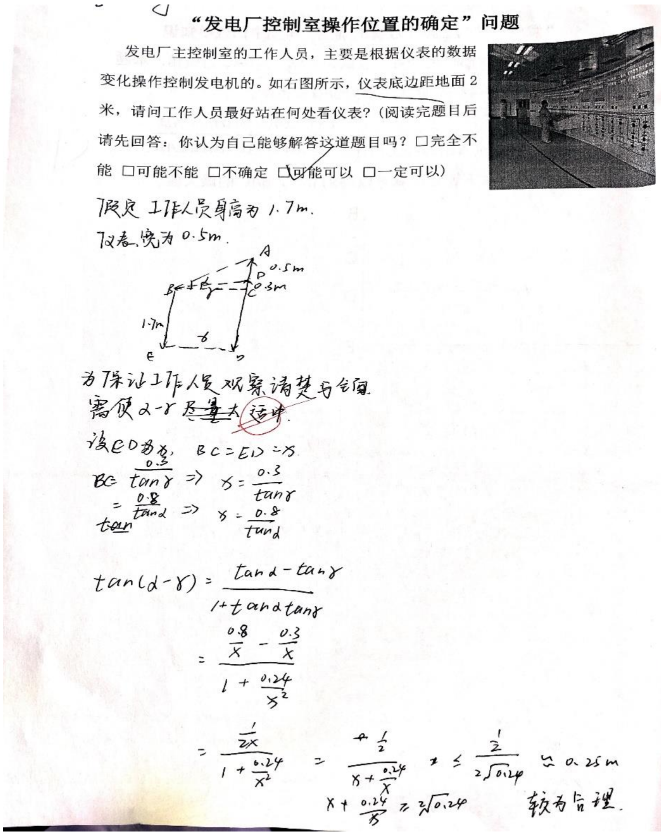

Figure 3: Li’s open-ended mathematical modeling answer

7. COMPARATIVE ANALYSIS OF MODELING PATHS BASED ON SSCS

The study used the open mathematical modeling coding table based on SSCS in Table 1. According to the performance of Wang and Li, the modeling steps of the two students in the process of solving the open mathematical modeling task - "Determination of the operating position of the power plant control room" were coded and visualized, as shown in Figure 3.

Observing the visualization of the modeling steps of the two students' open-ended mathematical modeling problem-solving process, first of all, the two students showed similar characteristics in the step of understanding the situation, that is, the path was interrupted, which shows that both students encountered certain difficulties in understanding the situation. Looking back on their solution process, in this step, they all had the same doubts about the conditions of reading the stem - they thought that the information given by the stem was insufficient. At first, they did not realize the open-ended nature of the question, and the researchers only told them that this question was different from the "application question" on the test paper. So after marking the useful information of the question, they tried to understand the "best place to stand" stated in the question. In this process, Li took a little more time than Wang. Next, after receiving the researcher's prompts about "openness" and "need to make assumptions", Wang immediately tried to simplify the question and began to try to build an abstract model in his mind, while Li started the "simplify and construct" step after a short path interruption after receiving the researcher's prompts. The emergence of this feature may be due to the different levels of previous mathematical experience and the speed of retrieval between the two students. Wang was able to simplify and construct immediately, which showed the continuity of his thinking. He could immediately try to call upon the schema that he thought was appropriate for the situation from his original cognitive system to assimilate and adapt. Compared with Wang, Li needed a certain amount of reaction time.

In the step of "simplifying and constructing", both Wang and Li successfully made necessary and realistic assumptions about the missing information in solving the problem. The most obvious difference between the two is that when the researchers reminded the two students that this modeling task actually requires finding the "largest observation angle", Wang had a step retreat in this step due to the researchers' hints on the understanding of the question; Li's performance was a path interruption, similar to continuing to think along the original thinking path after receiving instructions. Wang reread the conditions of the question stem and tried to understand the deep meaning behind the question. This feature reflects that Wang's thinking continuity when doing questions or thinking is better than Li's; it also reflects that Li's acceptance of hints is higher than Wang's. When the staff gave hints, Li chose to use the hints as known information, while Wang was more inclined to understand the hints as a way of thinking. This may also imply the difference in the listening habits of the two students in their daily mathematics learning.

In the step of "mathematicalization", both students used the method of "combining numbers and shapes" to express the simplified problem with intuitive sketches. Both students showed good continuity in their thinking in this step. Li spent slightly less time than Wang in this step. This may be because Wang was still trying to understand why the best position was the position with the "largest observation angle" when he was sketching, so he was slower in sketching on the test paper; while Li converted the real situation into a sketch according to the prompts, so he was faster in sketching on the test paper.

In the step of "establishing a mathematical model", there is a clear difference in the visualization of the modeling steps of the two: Wang's performance is very smooth, while Li has a path interruption and a step regression. Li thought that the prompt of "maximum observation angle" received at the beginning was biased and too absolute, so he changed "maximum" to "moderate". When he began to solve the mathematical model he established, he found that he could not continue, and there was a thought interruption at this time. After falling into silence, Li began to ask the researcher for help, so the researcher gave him a guiding teaching prompt, and Li began to rethink his assumptions and understanding of the problem, and there was a step regression to "simplify and construct". After re-understanding, Li was able to rebuild the mathematical model and solve it. Both of them finally completed this step, but Li took significantly longer time in this step than Wang due to the interruption of thinking and the regression of steps. In addition, in this step, Li's calculation results were wrong due to carelessness, while Wang performed better than Li in calculation ability.

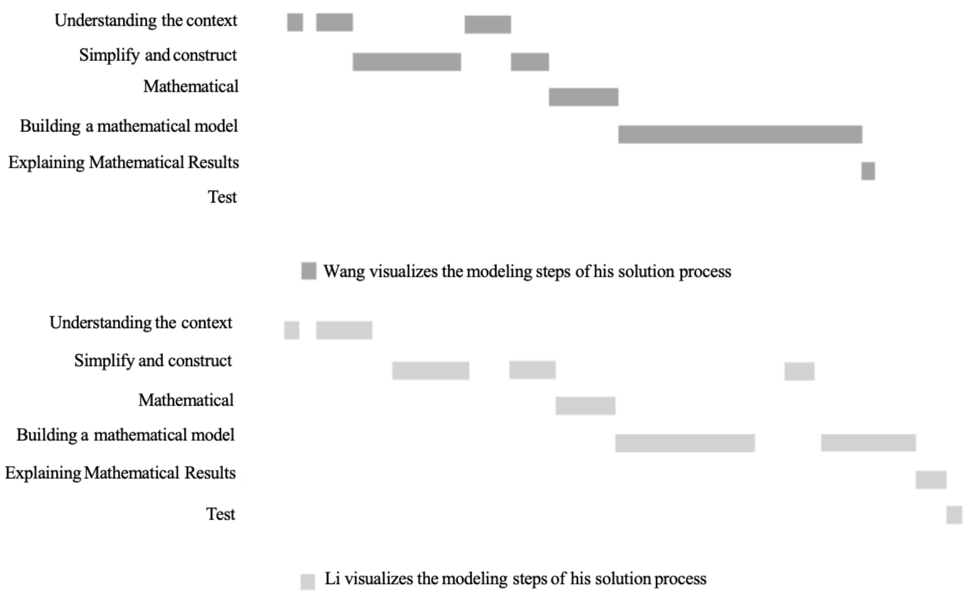

Figure 4: Visual comparison of the modeling steps of Wang and Li

Finally, the two students also showed significant differences in the two steps of "explaining mathematical results" and "verification". Li's performance in the step of "explaining mathematical results" was better than Wang's performance in these two steps: Li was able to explain his mathematical results based on real-life situations, estimate the mathematical results, and make his mathematical results closer to real-life scenarios; while Wang directly took the precise results obtained by his own calculations as the final answer. This shows that Li regards the open-ended mathematical modeling task as a problem to solve real-life situations, while Wang still cannot distinguish the open-ended mathematical modeling task from the "applied questions" in the math test paper. In addition, Li was able to have a "verification" step, that is, to verify whether his results were in line with the real situation, while Wang did not have a verification step. The difference in performance between the two in this link may still be due to the different levels of understanding of the two characteristics of the question: "openness" and "based on real-life situations".

After depicting the modeling paths of Wang and Li based on the SSCS modeling path diagram, we can more intuitively observe the differences in the cognitive level between the two when completing this open mathematical modeling task. In the figure, the numerical sequence on the arrow means the step of the student in the open mathematical modeling, showing the order of the student's thinking; "X" indicates a student's thinking interruption or a teaching prompt from the researcher; the arrows that appear repeatedly between the two boxes show the student's step back; the arrows represented by dotted lines reveal that the student has not actually completed the corresponding step, that is, the step was formally carried out, but not actually done, or there are certain problems in this step.

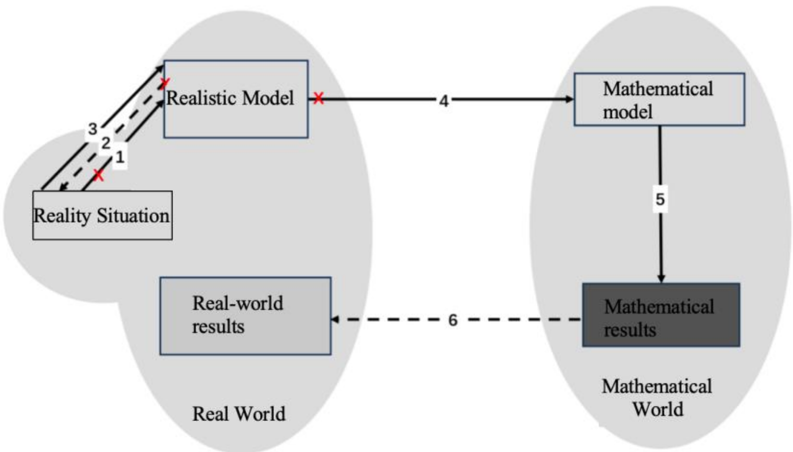

Figure 5: Wang’s open mathematical modeling path description

According to the content shown in Figure 5, Wang's open mathematical modeling based on SSCS path description has two "X"s in the step of understanding the real situation and simplifying it into a real model. At the same time, in his entire path, only this link has the phenomenon of step regression. This shows that when Wang is doing the open mathematical modeling task, understanding the real situation and constructing the real model from the situation is the most challenging step for him. This may be due to the "strangeness" caused by the first contact with open mathematical modeling, which corresponds to his answer to the self-efficacy scale question under this specific task. In addition, the "X" when converting the real model into a mathematical model represents that there are great difficulties in seeking a solution to this task. This is probably due to the fact that the mathematical problems contained in the open mathematical modeling task are not clear. This shows that Wang's ability to abstract mathematical problems from real situations is still lacking. The smooth progress from establishing a mathematical model to calculating mathematical results shows that Wang has a high level of application and calculation of mathematical knowledge, which may be inseparable from his previous mathematical experience. However, in the process of returning from the mathematical world to the real world, Wang's performance had certain problems, and the lack of reflection and verification of the results resulted in his final results still being unable to solve the problems in the real situation. This may indicate that the initial "sense of strangeness" did not disappear due to the smooth progress in the "mathematical world", which also caused a strong "sense of separation" between the "real world" and the "mathematical world" in his cognitive schema.

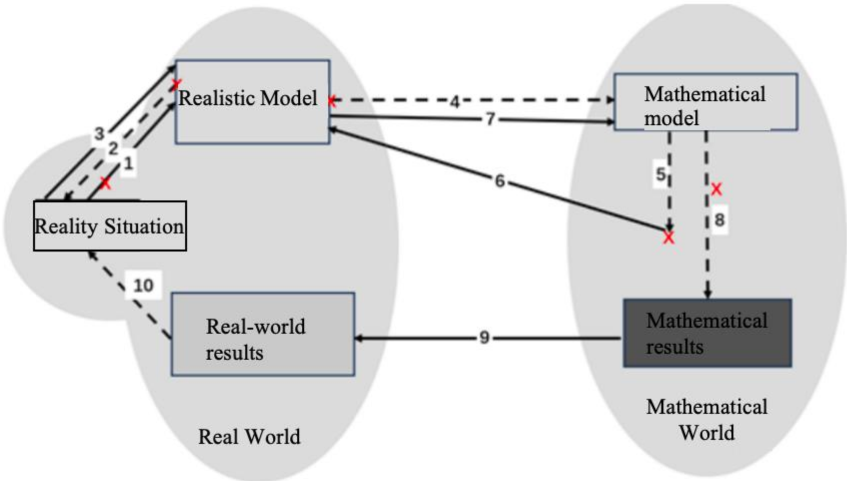

Figure 6: Li’s open mathematical modeling path description

According to the content shown in Figure 6, in the step of understanding the real situation and simplifying it into a real model based on the path characterization of SSCS in Li's open mathematical modeling, there are also two "X"s and one step retreat in the step of understanding the real situation and simplifying it into a real model according to the situation, which shows that this link is a challenge for both students who are doing the open mathematical modeling task for the first time. Secondly, the two "X"s between "real model" and "mathematical model" and the one step retreat between "real model" and "real model" in the mathematical operation link show that Li is lacking in the ability to abstract mathematical problems from the real situation and in understanding the teaching prompts of researchers, which shows that Li's thinking continuity in the mathematization step is not as good as Wang's performance in this aspect. In addition, in step 8, that is, the link of solving the mathematical model, Li remembered that he could use the content of the chapter on basic inequalities to solve the maximum value after receiving the teaching prompt, which shows that he is not flexible in calling the mathematical knowledge he has learned. In addition, he made errors in mathematical operations, which may be due to his lack of caution in the operation. This carelessness in calculation may also be the reason why he has a gap with Wang in his daily mathematical performance. Li's performance in converting mathematical results into real results is gratifying, which shows that his perception of real situations is better than Wang's. This also reflects that in Li's cognitive structure, the connection between the "real world" and the "mathematical world" is relatively good, and there is no deep "sense of separation". What is unsatisfactory is that although Li made a "relatively reasonable" test for the real results, this test was superficial and lacked systematic thinking. This is because, on the one hand, he did not give a set of test standards based on real situations, which made it seem that such a test did not have sufficient basis; on the other hand, he only judged his own calculation results and came to a "relatively reasonable" conclusion, but failed to consider whether the mathematical model and calculation steps he used were accurate and complete.

By comparing the characterizations of Wang and Li’s open mathematical modeling paths based on the SSCS modeling schema, we can find that: first, the "openness" of the task brought considerable challenges to both students. During this process, both students experienced certain path interruptions and step regressions; second, the process of constructing a realistic model and mathematizing it placed higher demands on the mathematical abstraction abilities of both students. Converting the "best position" to the "position with the largest observation angle" was a difficult point in understanding for both students; third, whether the progress in the "mathematical world" was smooth or not might imply the performance of the two students in mathematics. Wang's progress in the "mathematical world" was obviously smoother than that of Li; finally, students' daily mathematics performance did not necessarily show a positive correlation with their performance in open mathematical modeling. The "split" between the "real world" and the "mathematical world" might be the pain point and difficulty that prevented students from successfully solving open mathematical modeling problems.

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