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A Rational Review of Geometric Reasoning

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Abstract: ***This paper analyzes the necessity of geometric reasoning thinking for students' development by discussing the junior high school mathematics geometry course as an important carrier for the development of students' reasoning ability, summarizing relevant studies on geometric reasoning, and proposing the necessity and measures for the cultivation of geometric reasoning thinking.***

Keywords: Geometric Reasoning; Literature Review; Mathematics Education.

1. INTRODUCTION

With the rapid development of science and technology, we receive information from a wider and wider range of channels, in the face of increasing complex information, selecting information, judging the reasonableness and validity of the information is also gradually being valued by the people, and thus the ability to reason has become one of the necessary abilities for daily life and learning [1]. The cultivation of reasoning thinking has also become the key point of the curriculum reform in the new era. With the continuous advancement of the new curriculum reform, the focus of teaching has gradually shifted from the level of knowledge and skills to the aspect of core literacy, and the promotion of the development of students' core literacy is the key goal of mathematics teaching and even the entire school teaching process. Proposing six core literacy, geometry is not an isolated subject, but a way of thinking and problem solving that permeates the entire process of mathematics education, the Compulsory Education Mathematics Curriculum Standards [2] states that graphics and geometry, as one of the main lines running through the junior high school mathematics textbooks, is an important carrier for cultivating students' geometric intuition, reasoning ability and other core literacy in mathematics.

Mathematics with Core Literacy as the Main Objective teaching objectives focus on students' thinking development, and reasoning, as one of the basic forms of thinking, is an abstraction of the mathematical thinking process [3]. The curriculum standard divides the content of middle school mathematics into “number and algebra”, “graphics and geometry”, “statistics and probability”, and “synthesis and practice”. Among them, graphics and geometry is one of the main lines running through the whole middle school mathematics content, which is the carrier of cultivating students' mathematical thinking. Geometry courses have a profound logical structure, geometry learning requires the interaction of various abilities to promote the mastery of geometry content, and most of the geometry courses require reasoning ability, which is centered on reasoning ability to carry out learning and then promote problem solving, and in the process promote the development of habits of mind.

The geometry curriculum has a long history of development. At the beginning of the development of mathematics education, Euclid's Principia Geometric became the golden rule in mathematics education, and for a long time thereafter, geometry was Euclidean geometry, and in the early 1900s, the Klein-Bailey movement attacked Euclidean geometry, which had maintained a key position in the field of mathematics education. The Kline-Bailey movement in the early 20th century attacked Euclidean geometry, which remained key in mathematics education, and although it was later discontinued, it left a deep impact on the value of geometry, and contributed to the “New Numbers” movement, which, despite its failure, changed the way geometry was learned, and the “back to basics” movement of the 1980s was a sign of the decline of geometry. The “back-to-basics” movement in the 1980s was a stage in the decline of geometry, which led to further thinking about the value of geometry in mathematics education in the 1990s [4].

At this stage, the geometry curriculum in the field of secondary school mathematics is still dominated by Euclidean geometry, in which geometric proof problems take up a large part of the junior high school geometry curriculum, with the main focus on cultivating students' geometric reasoning thinking. However, due to the increasing difficulty of geometry problems, a large part of them cannot be directly found in life prototypes, which leads to students' acceptance difficulties, and then produce an aversion to learning. Due to the excessive pursuit of teaching progress in the current stage of the teaching process, ignoring the acceptance of students, one-sided demand for mastery of knowledge at the expense of the development of geometric thinking. And because of geometric thinking as a hidden thinking characteristic, not easy to accurately grasp, there is a teacher does not accurately grasp the level of students' geometric thinking and cause geometric teaching is too difficult, for example, students false understanding.

Therefore, to grasp the development level of students' geometric reasoning thinking and to promote the advancement of students' geometric reasoning thinking based on students' existing geometric reasoning thinking level is one of the focuses and difficulties of geometry teaching nowadays.

2. APPLIED RESEARCH ON GEOMETRIC REASONING

2.1 Theoretical Research on Learning Progression

In domestic research, scholars study and discuss the theoretical significance of geometric reasoning thinking, and propose that the development of geometric reasoning thinking is of great significance to the improvement of students' logical reasoning ability and problem-solving ability. In terms of foreign research, based on the Gestalt school and Piaget's theory of cognitive developmental stages, the Van Hil's put forward a theory of geometric thinking developmental levels, which divides students' geometric thinking development into five levels, i.e., visual, analytical, non-formal deduction, formal deduction, and rigor, and the five levels are gradual and reflect the progressive development of thinking [5].

Xie Meiyun [6] discusses the methods to improve students' geometric reasoning ability from the compulsory education mathematics curriculum standard, in which theorem teaching is the basis for improving students' geometric reasoning thinking, the problem setting of the theorem, the graph, the conclusion, and the role of the theorem must be deeply understood, and the different combinations of common geometric shapes can produce a wealth of variations, so the analysis of common geometric shapes is an effective means of solving complex geometric problems and improving students' learning confidence. The analysis of common geometric shapes is an effective means of solving complex geometric problems and enhancing students' confidence in learning.

2.2 Empirical Research on Learning Progression

Some scholars discuss geometric reasoning thinking based on empirical studies and provide practical suggestions for geometry teaching. Fuys [7] conducted an empirical study based on Van Hil's theory to analyze the characteristics of students' geometric thinking at each level, the potential for transitioning to the next level and the difficulties they face, and to analyze how teacher training and curricular arrangements can promote students' geometric thinking progress. Chang [8] discusses the geometric thinking level of junior high school students with the help of a multimedia software (GeoCAL). GeoCAL's interactive mode of presenting daily life objects and geometric shapes can help develop students' figurative reasoning, improve their understanding of the properties of geometric shapes, and have a significant impact on their reasoning of geometric concepts. Seah [9] et al. proposed geometric reasoning thinking as a bridge between linking mathematical language, visualization, and multiple representations, classified geometric reasoning thinking into five levels, included more detailed hierarchical divisions under several levels, compiled geometric reasoning test questions to discuss the geometric reasoning thinking levels of students in grades 7-10, and empirically analyzed to understand the progression of geometric reasoning thinking of secondary school students. Arnal Bailera [10] focused on level five in Van Hil's theory and used Delphi method to construct and validate a geometric reasoning model with respect to level five of Van Hil's geometric thinking level in four dimensions, namely, definition, classification, proof, and analysis. Lowrie [11] established an integrated mathematical model consisting of experience - linguistic - pictorial - symbolization - application (ELPSA), the Adams [12] explored the relationship between high school students' geometric reasoning and mathematical ability based on the ELPSA model, and made suggestions for mathematics classroom teaching based on the findings that spatial training can enhance students' level of mathematical achievement to a certain extent. Downton [13] used the qualitative research method of one-on-one task interviews to explore the learning experience of students' geometric reasoning using prisms as an example. Wilson [14] designed a generalized model for the assembly planning problem based on geometric reasoning to reach the geometric accessibility of the assembly tool. Miragliotta [15] proposed that geometric prediction is a fundamental and specific process of geometric reasoning, which is the process by which geometric shapes are manipulated and imagined while keeping their properties unchanged. Hohol [16] explained geometric reasoning from a cognitive perspective, arguing that geometric reasoning consists of both geometric representations and reasoning operations, and that explanations of geometric reasoning should not remain at the level of individualistic methodology, but should also take into account the stability of mathematical practice over time. Hamami [17] investigated the effect of counterexamples on geometric reasoning between points, lines, and circles, and the results of the study showed that increasing counterexample density (counterexample density) and decreasing counterexample distance enhanced subjects' geometric reasoning performance.

Cao Yiming and Ma Yingqiu [18] take the content selection of junior high school geometry textbooks as a starting point, compare the geometric content distribution characteristics of the six editions of mathematics textbooks in six regions, the geometric content selection of mathematics textbooks should be continuously explored in terms of depth, breadth, and presentation, and analyze the similarities and differences of geometric reasoning in junior high school textbooks in different regions through empirical research, and conduct comparative research in terms of presentation and depth hierarchy, and conclude that junior high school textbooks geometric reasoning should Focus on gradualism, and the presentation method is simple and concise. Based on the classification of conventional reasoning and creative reasoning, Shengqing He and Chunxia Qi [19] selected geometry as the best content for testing students' mathematical reasoning ability, prepared a geometric reasoning test paper, and conducted a large-scale test on the geometric reasoning ability of eighth-grade students in Z city in terms of conventional and creative reasoning, analyzed the eighth-grade students' geometric reasoning ability in terms of mathematics academic level, content area and gender, and analyzed the eighth-grade students' geometric reasoning ability in terms of mathematics academic level, content area and gender. The study analyzed the variability of eighth-grade students' geometric reasoning ability in terms of mathematics academic level, content area, and gender, and recommended building a bridge between "intuition" and "reasoning" and strengthening the integration of different content areas. Li Hongting [20] pointed out that mathematics teaching should focus on the cultivation of students' mathematical reasoning ability, which is the core issue of mathematics education. In addition, Li Hongting also investigated students' geometric reasoning and characteristics through empirical research based on the theory of children's intellectual development and the theory of geometric thinking of Van Hier, and found that language affects the progression of students' geometric reasoning level, which includes textual, graphic and symbolic language and the transformation between them. The study found that language affects the progression of students' geometric reasoning level, including textual language, graphic language, symbolic language and the conversion between them. Wang Kuanming [21] investigates the differences in students' geometric reasoning ability based on Van Hil's geometric thinking theory, determines the level of geometric thinking that students are at, compares the thinking characteristics of students with different geometric reasoning abilities, and then proposes practical suggestions for secondary school mathematics teaching, the key to improving students' geometric reasoning ability is the mastery of holistic structural relationships, and organizes teaching according to students' geometric reasoning thinking level. Guan Hongyan and Zhou Chao [22] took math teachers as the research object and Usiskins test paper as the research tool, focusing on teachers' geometric thinking level and providing reference suggestions for pre-service teacher education and in-service teacher training.

3. THE IMPORTANCE AND DEVELOPMENT OF GEOMETRIC REASONING THINKING

Geometric reasoning thinking is a thinking process that uses geometric shapes as a carrier to deduce unknown conclusions from known conditions through logical deduction, spatial imagination and problem solving. It is not only a core competency in mathematics, but also the key to developing logical thinking, creativity and interdisciplinary applications. Geometric reasoning refers to the thinking process of observing and analyzing the properties and interrelationships of geometric shapes (points, lines, surfaces, and bodies), and applying definitions, axioms, theorems, and logical rules to derive new conclusions from known conditions. Its nature is "from concrete to abstract, and then from abstract to concrete" cycle, emphasizing rigorous, systematic and creative. Geometric reasoning emphasizes logical rigor: each step should have a strict basis and conditions; at the same time, geometric reasoning is closely related to problem solving, and therefore emphasizes the problem-oriented: problem-driven, thinking activities throughout.

Geometric reasoning thinking requires first of all the construction of a logically rigorous knowledge system, the establishment of a logical chain between the concepts, the ability to conceptual understanding and analysis, such as the relationship between "isosceles triangle" and "equilateral triangle", "rectangle", "rhombus", 'parallelogram', "square" and their respective concepts and their connections; for mathematical theorems to establish a dynamic understanding process, and to establish a dynamic understanding of the theorem, such as "rectangle", "rhombus", "parallelogram" and "square" and their connections; to establish a dynamic process of understanding mathematical theorems. It is important to establish a dynamic process of understanding mathematical theorems, and to understand the sufficient and necessary conditions in theorems. Secondly, we emphasize the rigor and scientificity of logic, which is the core of mathematical thinking. Therefore, the use of assessment tools to express students' logical reasoning process explicitly helps to test the characteristics and difficulties of students' reasoning process, so as to improve students' geometric reasoning ability.

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